TRINITY COLLEGE

Year 11 Specialist Mathematics Units 1,2
Test 5 2021

Section 1 Calculator Free
Matrices

STUDENT'S NAME SOLNS

DATE: Monday 30 August **TIME:** 25 minutes **MARKS:** 24

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (7 marks)

Consider the matrices below.

$$A=\begin{bmatrix}1 & -5\\ -1 & 3\end{bmatrix} \quad B=\begin{bmatrix}4 & 0 & -2\\ 0 & 1 & 6\end{bmatrix} \quad C=\begin{bmatrix}7 & -9\\ 0 & 5\\ 4 & -1\end{bmatrix} \quad D=\begin{bmatrix}6 & -4 & 0\\ 2 & 11 & -3\end{bmatrix}$$

2×2 $\quad 2\times3$ $\quad 3\times2$ $\quad 2\times3$

(a) Determine $3D-2B$. [2]

$$\begin{bmatrix}10 & -12 & 4\\ 6 & 31 & -21\end{bmatrix}$$

(b) Matrices ABC can be multiplied, in that order to form another matrix. Give two other possible orders of multiplying matrices A, B and C that will form another matrix. [2]

BCA

CAB

(c) Matrix W is such that $AW-B=D$. Determine matrix W. [3]

$$AW = \begin{bmatrix}4 & 0 & -2\\ 0 & 1 & 6\end{bmatrix} + \begin{bmatrix}6 & -4 & 0\\ 2 & 11 & -3\end{bmatrix}$$

$$= \begin{bmatrix}10 & -4 & -2\\ 2 & 12 & 3\end{bmatrix}$$

$$W = -\frac{1}{2}\begin{bmatrix}3 & 5\\ 1 & 1\end{bmatrix}\begin{bmatrix}10 & -4 & -2\\ 2 & 12 & 3\end{bmatrix}$$

$$= \begin{bmatrix}-20 & -24 & -\frac{9}{2}\\ -6 & -4 & -\frac{1}{2}\end{bmatrix}$$

2. (5 marks)

Consider the matrices below.

$$A = \begin{bmatrix} k & 5 \\ k-1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & -5 \\ 1-k & k \end{bmatrix}$$

(a) Determine the value/s of k if A^{-1} is singular. [2]

$$7k - 5(k-1) = 0$$
$$2k + 5 = 0$$
$$k = -\frac{5}{2}$$

(b) Determine the value/s of k if matrices A and B are the inverse of each other. [3]

$$\begin{bmatrix} k & 5 \\ k-1 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ 1-k & k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7k + 5(1-k) & 0 \\ 0 & -5(k-1) + 7k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7k + 5 - 5k = 1 \qquad -5k + 5 + 7k = 1$$
$$2k = -4 \qquad 2k = -4$$
$$k = -2 \qquad k = -2$$

3. (7 marks)

Let A be a non-singular square matrix such that $4A^2 + 6A = I$ where I is the identity matrix.

(a) Prove that $2A^3 - 5A + 0.75I = 0$ where O is the zero matrix. [4]

$4A^2 + 6A = I$

$4A^2 = I - 6A$

$A^2 = \frac{I}{4} - \frac{3A}{2}$

$$
\begin{aligned}
LHS &= 2A\,A^2 - 5A - 0.75I \\
&= 2A\left(\frac{I}{4} - \frac{3A}{2}\right) - 5A - 0.75I \\
&= \frac{A}{2} - 3A^2 - 5A - 0.75I \\
&= \frac{A}{2} - 3\left(\frac{I}{4} - \frac{3A}{2}\right) - 5A - 0.75I \\
&= \frac{A}{2} - \frac{3I}{4} + \frac{9A}{2} - 5A - 0.75I \\
&= 0 \\
&= RHS
\end{aligned}
$$

(b) Hence determine the value of c such that $cA^{-1} = 10I - 4A^2$. [3]

$$2A^3 - 5A + 0.75I = 0$$

$(\times A^{-1})$ $$2A^2 - 5I + 0.75A^{-1} = 0$$

$$0.75A^{-1} = 5I - 2A^2$$

$$1.5A^{-1} = 10I - 4A^2$$

$$c = 1.5$$

4. (5 marks)

Given $P^3 = \begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = Q^{-1}$ determine $(ABQ)^3$.

$$(ABQ)^3$$

$$= ABQ\,ABQ\,ABQ$$

$$= AB\cancel{Q}\cancel{Q^{-1}}B\cancel{Q}\cancel{Q^{-1}}BQ$$

$$= AB^3Q$$

$$= \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -29 \\ 11 & -17 \end{bmatrix}$$

TRINITY COLLEGE

Year 11 Specialist Mathematics Units 1,2
Test 5 2021

Section 2 Calculator Assumed
Matrices

STUDENT'S NAME ______________________________

DATE: Wednesday 31st March **TIME:** 25 minutes **MARKS**: 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (3 marks)

Determine the matrix that transforms $A(-1,5)$ to $A'(7,14)$ and $B(4,2)$ to $B'(-6,32)$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 14 & 32 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 14 & 32 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 5 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -2 & 1 \\ 6 & 4 \end{bmatrix}$$

6. (6 marks)

A pair of linear equations in x and y is determined by

$$\begin{bmatrix} 3 & -4 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -34 \\ -44 \end{bmatrix}$$

(a) Use an inverse matrix to determine x and y. [3]

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} -34 \\ -44 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

(b) Hence solve the equation $\begin{bmatrix} 3 & -4 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x+5 \\ x^2+1 \end{bmatrix} = \begin{bmatrix} -34 \\ -44 \end{bmatrix}$ [3]

$$\begin{bmatrix} x+5 \\ x^2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

$x+5 = 2$ $\qquad$ $x^2+1 = 10$

$x = -3$ $\qquad$ $x = \pm 3$

$x = -3$

7. (5 marks)

Let S be a shear transformation matrix of factor $\frac{1}{2}$ parallel to the x-axis.

(a) State matrix S. [1]

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$

(b) What does S^{-1} represent? [2]

$$S^{-1} = \begin{bmatrix} 1 & -0.5 \\ 0 & 1 \end{bmatrix}$$

SHEAR PARALLEL TO x - AXIS SCALE FACTOR $-\frac{1}{2}$

(c) Show mathematically that if ANY shear matrix is applied to ANY geometric figure, the area of the image will always be equal to the area of the original figure. [2]

DETERMINANTS OF ALL SHEARS = 1

$\therefore$ AREA WILL NOT CHANGE.

8. (5 marks)

All the points on the line $y = 2x - 5$ are transformed by the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Determine the equation of the image of the line.

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} k \\ 2k-5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$k + 2(2k-5) = x$$
$$k + 4k - 10 = x$$
$$5k - 10 = x$$
$$k = \frac{x+10}{5}$$

$$y = k$$
$$y = \frac{x+10}{5}$$

9. (10 marks)

The parallelogram $PQRS$ with coordinates $P(2,1)$, $Q(5,2)$, $R(6,7)$ and $S(3,6)$ is transformed to parallelogram $P'Q'R'S'$ with coordinates $P'(1,-2)$, $Q'(2,-5)$, $R'(7,-6)$ and $S'(6,-3)$.

(a) Describe the geometrical effect of the transformation and give the appropriate transformational matrix. [3]

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ 90° CLOCKWISE ROTATION

Parallelogram $P'Q'R'S'$ is then transformed by the matrix $\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$ to $P''Q''R''S''$.

(b) Determine the coordinates of P'', Q'', R'' and S''. [2]

$$\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 7 & 6 \\ -2 & -5 & -6 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 14 & 12 \\ 6 & 15 & 18 & 9 \end{bmatrix}$$

(c) If the area of $PQRS$ is 12 units^2, calculate the area of $P''Q''R''S''$. [2]

$$\begin{vmatrix} 2 & 0 \\ 0 & -3 \end{vmatrix} = 6$$

$$\text{AREA} = 6 \times 12 = 72$$

(d) Determine a single matrix which will map parallelogram $P''Q''R''S''$ to parallelogram ~~ABCD~~. PQRS [3]

$$\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}^{-1} = -\frac{1}{6}\begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$$